Entanglement and decoherence within neutron interferometry

Katharina Durstberger-Rennhofer, Georg Sulyok, Yuji Hasegawa, Helmut Rauch
Institute of Atomic and Subatomic Physics, Vienna University of Technology

Introduction

Fundamental quantum properties like quantum coherence and entanglement are among the most interesting features of quantum mechanics. They form the basis for the new, fast developing fields of quantum information and computation.

Neutron interferometry is a very proper tool to study quantum mechanical behaviour. Entanglement between different degrees of freedom (e.g., spin and path) can be established. By varying magnetic fields decoherence and dephasing is introduced and studied.

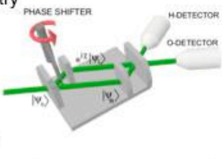
We investigate different kinds of entanglement for the neutron system and mechanisms for decoherence and dephasing.

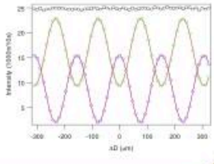
Neutron interferometry (since 1974)

- matter wave interferometry
- self interference of neutrons
- perfect single silicon crystal interferometer
  topological identical to Mach-Zehnder interferometer
  beam separation ~ 5 cm
- intensity oscillations in both detectors

$I_O \propto \frac{1}{2}(1+\cos\chi)$

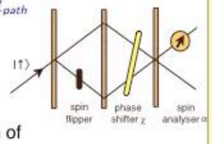
$I_H \propto \frac{1}{2}(1-\cos\chi)$



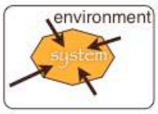


Entanglement for single neutrons

- bipartite Hilbert space
  $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 = \mathcal{H}_{spin} \otimes \mathcal{H}_{path}$
- entangled state
  $|\Psi\rangle = \frac{1}{\sqrt{2}}(|\Uparrow\rangle \otimes |I\rangle + |\Downarrow\rangle \otimes |II\rangle)$
  entanglement between spin and path degree of freedom
- Bell inequality: combination of expectation values
  $S(\alpha,\alpha',\chi,\chi') = |E(\alpha,\chi) - E(\alpha,\chi')| + |E(\alpha',\chi) + E(\alpha',\chi')| \le 2$
- QM violates BI
  $S_{max} = 2\sqrt{2} \sim 2.828 > 2$
  $S_{exp,photon} = 2.73 \pm 0.02$ $S_{exp,neutron} = 2.051 \pm 0.019$
- with neutrons: test of contextuality
  with photons: test of locality



Decoherence theory



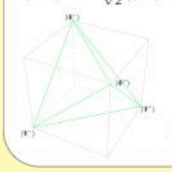
- interaction between system S and environment E
- dissipation: energy flow
  decoherence: loss of interference
- Kraus operators
  nonunitary evolution by "jump operators"
  $\rho_0 \mapsto \rho_t = \sum_i M_i \rho_0 M_i^\dagger$ with $\sum_i M_i^\dagger M_i = \mathbb{1}$
- Lindblad master equation
  $$\underbrace{\frac{\partial}{\partial t}\rho_t = -i[H,\rho_t]}_{\text{Liouville-von Neumann equation}} - \underbrace{\mathcal{D}(\rho_t)}_{\text{nonunitary part}}$$
  Dissipator $\mathcal{D}(\rho_t) = \lambda(\rho_t - \sum_k P_k \rho_t P_k)$
  Lindblad generators $P_k^2 = P_k$
- examples for decoherence channels for 1 qubit:
  phase damping channel, depolarizing channel

$M_0 = \sqrt{1-\frac{3p}{4}}$ - $M_2 = \sqrt{\frac{p}{4}}\,\sigma_y$ $M_0 = \sqrt{1-\frac{p}{2}}$ -

$M_1 = \sqrt{\frac{p}{4}}\,\sigma_x$ $M_3 = \sqrt{\frac{p}{4}}\,\sigma_z$ $M_1 = \sqrt{\frac{p}{2}}\,\sigma_z$

Spin geometry picture

- 1 qubit:
  $\rho = \frac{1}{2}(\mathbb{1} + \vec{r}\cdot\vec{\sigma})$
- 2 qubits:
  $\rho = \frac{1}{4}(\mathbb{1}\otimes\mathbb{1} + \vec{m}\,\vec{\sigma}\otimes\mathbb{1} + \vec{n}\,\mathbb{1}\otimes\vec{\sigma} + c_{ij}\,\sigma_i\otimes\sigma_j)$
  local parameters nonlocal correlations
- example: Bell states
  $|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ $\vec{m} = \vec{n} = 0$
  $c^\Psi = \mathrm{diag}(\pm1,\pm1,-1)$
  $|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ $c^\Phi = \mathrm{diag}(\pm1,\mp1,1)$

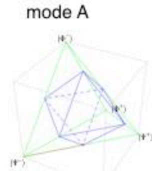


tetrahedron of possible states

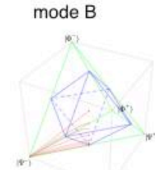
octahedron of separable states

Geometric picture of deco modes

mode A



mode B

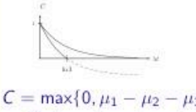


Decoherence modes

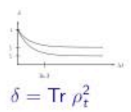
- master equation
  $\frac{\partial}{\partial t}\rho_t = -\mathcal{D}(\rho_t) = -\lambda(\rho_t - \sum_k P_k\rho_t P_k)$
  decoherence parameter λ determines strength of interaction
- initial state
  $$\rho_0 = |\Psi^-\rangle\langle\Psi^-| = \frac{1}{2}\begin{pmatrix}0&0&0&0\\0&1&-1&0\\0&-1&1&0\\0&0&0&0\end{pmatrix}$$
- **decoherence mode "E⊗E"**
  projectors on the eigenbasis of the system
  $P_k = P_k^{(1)} \otimes P_k^{(2)}$
  final state
  $$\rho_t = \frac{1}{2}\begin{pmatrix}0&0&0&0\\0&1&-e^{-\lambda t}&0\\0&-e^{-\lambda t}&1&0\\0&0&0&0\end{pmatrix}$$
  ⇨ dying out of the off-diagonal elements
  Kraus operators: phase damping channel
  $M_0 \propto \mathbb{1}\otimes\mathbb{1}$ $M_1 \propto \mathbb{1}\otimes\sigma_z$
  $M_2 \propto \sigma_z\otimes\mathbb{1}$ $M_3 \propto \sigma_z\otimes\sigma_z$
- **decoherence mode "R⊗E"**
  projectors on rotated basis in one subsystem
  $P_k = UP_k^{(1)}U^\dagger \otimes P_k^{(2)}$
  final state
  $$\rho_t = \frac{1}{4}\begin{pmatrix}1-e^{-\lambda t}&0&0&0\\0&1+e^{-\lambda t}&-2e^{-\lambda t}&0\\0&-2e^{-\lambda t}&1+e^{-\lambda t}&0\\0&0&0&1-e^{-\lambda t}\end{pmatrix}$$
  ⇨ dying out of the off-diagonal elements
  ⇨ and evolution to totally mixed state (redistribution of diagonal elements)
  Kraus operators: phase damping & depolarizing channel
  $M_0 \propto \mathbb{1}\otimes\mathbb{1}$ $M_1 \propto \mathbb{1}\otimes\sigma_z$
  $M_2 \propto \sigma_x\otimes\mathbb{1}$ $M_3 \propto \sigma_x\otimes\sigma_z$

Entanglement - mixedness

concurrence C



mixedness δ



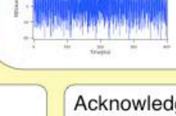
$C = \max\{0, \mu_1 - \mu_2 - \mu_3 - \mu_4\}$ $\delta = \mathrm{Tr}\,\rho_t^2$

μ_i^2 eigenvalues of matrix

$R = \rho_t(\sigma_y\otimes\sigma_y)\rho_t^*(\sigma_y\otimes\sigma_y)$

Gaussian magnetic noise fields

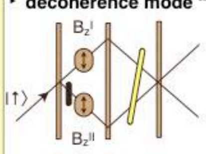
typical distribution of Gaussian white noise



histogram of amplitude distribution

Experimental realisation of decoherence modes for neutrons

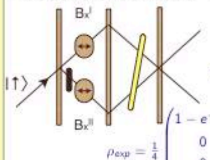
- decoherence = randomly fluctuating magnetic fields
- every neutron „feels" separately a different but constant field (quasistatic approximation)
  Gaussian distribution of field strengths $P(\alpha) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{\alpha^2}{2\sigma^2}}$
- magnetic field induces unitary transformation
  $U(\alpha) = e^{i\frac{\alpha}{2}\vec{n}\cdot\vec{\sigma}}$ with $\alpha = 2\mu Bt = \omega_L t$
- nonunitary evolution due to ensemble average
  $$\rho \longrightarrow \rho' = \int \underbrace{U(\alpha)\,\rho\,U^\dagger(\alpha)}_{\rho(\alpha)}\,P(\alpha)d\alpha$$
- **decoherence mode "E⊗E"**



$\rho_{exp} = \int \rho(\alpha,\beta)\,P(\alpha)\,P(\beta)d\alpha\,d\beta$

$$\rho_{exp} = \frac{1}{2}\begin{pmatrix}0&0&0&0\\0&1&-e^{-\frac{\sigma^2}{4}}&0\\0&-e^{-\frac{\sigma^2}{4}}&1&0\\0&0&0&0\end{pmatrix}$$

- **decoherence mode "R⊗E"**



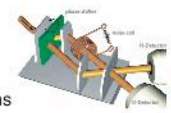
$\rho_{exp} = \int \rho(\alpha,\beta)\,P(\alpha)\,P(\beta)d\alpha\,d\beta$

$$\rho_{exp} = \frac{1}{4}\begin{pmatrix}1-e^{-\frac{\sigma^2}{2}}&0&0&0\\0&1+e^{-\frac{\sigma^2}{2}}&-2e^{-\frac{\sigma^2}{2}}&0\\0&-2e^{-\frac{\sigma^2}{2}}&1+e^{-\frac{\sigma^2}{2}}&0\\0&0&0&1-e^{-\frac{\sigma^2}{2}}\end{pmatrix}$$

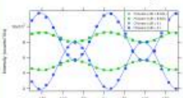
- **result** $\boxed{\lambda t \propto \sigma^2}$
  decoherence parameter is related to the standard deviation of the fluctuations

Noise induced decoherence

- experimental setup
  ⇨ unpolarized neutron beam
  ⇨ quasistatic approximation valid (tof 20.7μs through coil)

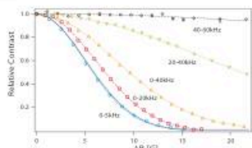


- measured intensity oscilations



⇨ reduction of the measured contrast

- contrast
  ⇨ in depencence on the width of the fluctuations



$$C = \int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\Delta B}e^{-\frac{B^2}{2(\Delta B)^2}}\cos\left(\frac{\mu}{\hbar}\frac{l}{v}B\right)dB = \exp\left\{-\frac{1}{2}\left(\frac{\mu l}{\hbar v}\Delta B\right)^2\right\}$$

References

(1) Y. Hasegawa, R. Loidl, G. Badurek, M. Baron, and H. Rauch, Nature **425**, 45 (2003).
(2) R. A. Bertlmann, K. Durstberger, and Y. Hasegawa, Phys. Rev. A **73**, 022111 (2006).
(3) K. Durstberger, Phys. Lett. A **372**, 1789 (2008).
(4) G. Sulyok, Y. Hasegawa, J. Klepp, H. Lemmel, and H. Rauch, submitted to Phys. Rev. A (2010).

Acknowledgement

This work has been supported by the Austrian Science Foundation, FWF. K. D.-R. acknowledges funding of her work by a Hertha Firnberg Position (T389-N16) and G. S. was funded by the doctoral program on Complex Quantum Systems of the Austrian Science Foundation. The measurements were done at instrument S18 at the Institut Laue Langevin (ILL) in Grenoble.